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# Smart Coordinate System for Scandinavia Countries

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## ABSTRACT

The development of technologies in the field of geodesy and map projections is important for the coordinate system used in surveying works and geographic information system "GIS". This paper presents a system of coordinates by harmonic equations projection "the united projections" that has five projections (Mercator, Lambert, Russell, Lagrange, and the compound projections) in one zone coordinate system. The theory of the projections by a harmonic equation as well as Lagrange projection has eight direct algorithms defined by Professor Vladimir podshivolev 1998. These algorithms have some difficulties and very complicated method.

A new direct algorithms for all five projections have been presented, as well as a new coordinate system by compound projection for Scandinavian countries. Also the distortion scale factor for measuring distances in Sweden by the smart main system for some cities have been discussed.

**Keywords:** system, compound projection, coordinates, algorithms, harmonic equations.

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## 1. INTRODUCTION

The theory of united projections was introduced by Prof. Vladimir Podshivalov in 1998, it was aimed for special cases (construction system coordinates for GIS of countries by 12\*12 degrees long and the width of the zone); In 2009-2012 Dr Akresh found the general law for indirect algorithms for five projections, general law for direct algorithms of Russell projection and also in Lagrange projection. The zone here is larger than before and reaches 24\*24 degrees.

The theory of united projections constructs 5 projections, each projection also has a local system for big cities, and this system has an advantage in decreasing of distances distortion and very easy way to go back to the main of coordinate system.

All countries after 50 degrees in latitude north or south faces some difficulties in constructing a good coordinate system.

Universal transverse Mercator projection covers Sweden in 4 zones “32, 33, 34, 35”, and these zones have too many problems because all zones are conform, also has high distortion in the scale factor.

## 2. METHODOLOGY

The methodologies in new map projection have standard parallels for any zones. Scientist Grave Chipeshiv 1845 (Podshivalov, 1998&Yury,2007) , proposed creating a new projection by two projections with two new scale factors  $k_1$  and  $k_2$  in which  $k_1+k_2=1$ .

This study uses projections of Lambert and Mercator together for creating new algorithm, and other steps with the same method uses four projections by harmonic equations named as compound projection; it has special properties for distortion of scale factor, where all projections (Mercator, Lambert, Russell and Lagrange) haven't these properties.

Compound projection algorithms created by direct algorithms Lambert and Mercator and has a new scale factor, follows these algorithms (Akresh, 2012&Morozov, 1979) .

First: direct algorithms of Mercator projection

$$C_1 = \frac{m_0 \cdot c \cdot \cos B_0}{V}, \quad C_2 = -\frac{C_1 \cdot \sin B_0}{2}, \quad C_3 = \frac{C_1 \cdot \cos^2 B_0}{6} (\tan^2 B_0 - V^2),$$

$$C_{14} = \frac{C_1 \sin B_0 \cos^{12} B_0}{479001600} (248010751 - 2137192389 \tan^2 B_0 + 3002137335 \tan^4 B_0 - 1081702420 \tan^6 B_0 + 73802835 \tan^8 B_0 - 398574 \tan^{10} B_0 + \tan^{12} B_0); \quad (1)$$

Second: direct algorithms of Lambert projection

$$C_j = \frac{C_1}{j!} (-1)^{(j-1)} (\sin B_0)^{(j-1)}, \dots, J = 1, 2, \dots, n.$$

$$C_1 = C_1, C_2 = -\frac{C_1}{2} \sin B_0, C_3 = \frac{C_1}{6} \sin^2 B_0, C_4 = -\frac{C_1}{24} \sin^3 B_0, \dots \quad (2)$$

The compound projection uses two scale factors and the sum of them must to be equal one ; If  $k_1 = 0.5$  ,  $k_2 = 0.5$  a projection of Russell will created, if other new two scale factors values were used then different geometric figures will be created “new models for compound projection”. For choosing the two scale factors the method of adjustment by least square method observation were used, firstly the following equations were used ((Podshivalov, 1998&Yury, 2007)).

$$0 + k_1 + k_2 = 1$$

$$m + k_1 \left( -\frac{\Delta X_1^2}{2m_0 R_0^2} \right) + k_2 \left( -\frac{\Delta Y_1^2}{2m_0 R_0^2} \right) = m_0$$

$$m + k_1 \left( -\frac{\Delta X_2^2}{2m_0 R_0^2} \right) + k_2 \left( -\frac{\Delta Y_2^2}{2m_0 R_0^2} \right) = m_0$$

$$m + k_1 \left( -\frac{\Delta X_3^2}{2m_0 R_0^2} \right) + k_2 \left( -\frac{\Delta Y_3^2}{2m_0 R_0^2} \right) = m_0$$

Second step using method of observation only:

$$Q_e = AQA^T, \dots, Q = 1$$

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ -\frac{\Delta X_1^2}{2m_0 R_0} & -\frac{\Delta X_2^2}{2m_0 R_0} & -\frac{\Delta X_3^2}{2m_0 R_0} & 1 \\ -\frac{\Delta Y_1^2}{2m_0 R_0} & -\frac{\Delta Y_2^2}{2m_0 R_0} & -\frac{\Delta Y_3^2}{2m_0 R_0} & 1 \end{bmatrix}, K = Q_e^{-1} F, F = \begin{bmatrix} m_0 & m_0 & 1 \end{bmatrix},$$

$$V = A^T K, V = \begin{bmatrix} m & k_1 & k_2 \end{bmatrix}. \quad (3)$$

### 3. CASE STUDY

The area was chosen near the North Pole; Scandinavian countries. The coordinate system of Sweden was tested using new coordinates system by compound projection with parabola shape. The results obtained then, compared with universal transverse Mercator UTM6 results. The parameters that used are  $k_1 = -0.15$ ,  $k_2 = 1.15$ , and main scale factor  $m_0 = 1.000000$ , standard parallel  $61^\circ 21' N$ , center meridian  $16^\circ 30' E$  and  $750000.00$  m for coordinate of “y” in center meridian edges of zone ( $18^\circ X 18^\circ$ ) using WGS84 and given the name smart main coordinates system (fig. 1).

This study applied on some cities in Sweden “Stockholm, Orebro, Malmo”, using a special scale factor for each of them. If the relationship between Local and main system were used, then the following equations can be used. All results listed in tables (1, 2, 3, 4, 5, 6).

$$\frac{dx_{local}}{dx_{main}} = \frac{m_{local}}{m_{main}}, \quad X_{local} = X_0 + dx_{local}, \quad X_{main} = X_0 + dx_{main} \quad (4)$$

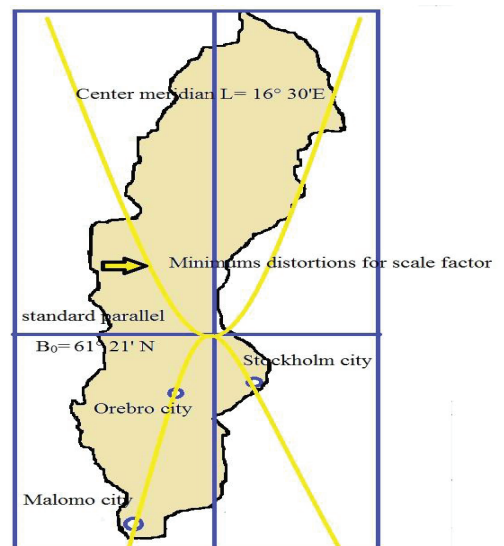


Fig (1). Study Area of Sweden

**Table-1 Stockholm city – comparison between UTM and Compound projection results**

Ellipsoid parameters WGS 84 a= 6378137.00m, b=6356752.314m		
Distance from equator to standard parallel (Projection of compound) $X_0=6804494.7527$ m		
Scale factor for projections	0.9996	K1=-0.15 k2=1.15
Projections	UTM –Mercator zone 33 , L=15°00' E	Projection of compound standard parallel $B_0=$ 61° 21' N Center meridian L= 16° 30'E
Geographic coordinates lat.1	N «00<22 59°	
Geographic coordinates log.1	E «30<49 17°	
Triangular coordinates x1	6584288.261	6584268.963
Triangular coordinates y1	660542.610	825333.989
Scale factor point 1	0.9999159	0.99999203
Geographic coordinates lat.2	N «00<22 59°	
Geographic coordinates log.2	E «00<55 17°	
Triangular coordinates x2	6584512.949	6584375.947
Triangular coordinates y2	665749.827	830545.326
Scale factor point2	0.9999367	1.00000348
Distance for plane	5212.062	5212.435
Distance for Geo. Problems	5212.446	
Relative scale for distances	1/13574	1/473866

**Table-2 Stockholm city – comparison between UTM and Compound projection results**

<b>Ellipsoid parameters WGS 84 a= 6378137.00m, b=6356752.314m</b>		
<b>Distance from equator to standard parallel (Projection of compound)</b> <b><math>X_0=6804494.7527</math> m</b>		
<b>Scale factor for projections</b>	<b>0.9996</b>	<b>K1=-0.15 k2=1.15</b>
Projections	UTM –Mercator zone 33 , L=15°00' E	Projection of compound standard parallel $B_0=61^\circ$ 21' N Center meridian L= 16° 30'E
Geographic coordinates lat.3	N «00<20 59°	
Geographic coordinates log.3	E «00<55 17°	
Triangular coordinates x3	6580803.372	6580663.402
Triangular coordinates y3	665912.424	830624.075
Scale factor point 3	0.99993739	1.00000071
Geographic coordinates lat.4	N «00<24 59°	
Geographic coordinates log.4	E «00<55 17°	
Triangular coordinates x4	6588222.540	6588088.521
Triangular coordinates y4	665587.173	830466.544
Scale factor point4	0.9999361	1.00000621
Distance for plane	7426.294	7426.790
Distance for Geo. Problems	m 7426.764	
Relative scale for distances	1/15801	1/285645

Table-3 Orebro city – comparison between UTM and Compound projection results

<b>Ellipsoid parameters WGS 84 a= 6378137.00m, b=6356752.314m</b>		
<b>Distance from equator to standard parallel (Projection of compound) <math>X_0=6804494.7527</math> m</b>		
<b>Scale factor for projections</b>	<b>0.9996</b>	<b>K1=-0.15 k2=1.15</b>
Projections	UTM –Merca- tor  zone 33 , L=15°00' E	Projection of compound  standard parallel $B_0=$ 61° 21' N  Center meridian L= 16° 30'E
Geographic coordinates lat.1	N «00<16 59°	
Geographic coordinates log.1	E «30<09 15°	
Triangular coordinates x1	6569756.837	6573150.181
Triangular coordinates y1	509026.175	673494.840
Scale factor point 1	0.9996009	0.9999855
Geographic coordinates lat.2	N «00<16 59°	
Geographic coordinates log.2	E «30<13 15°	
Triangular coordinates x2	6569767.765	6573075.793
Triangular coordinates y2	512826.662	677296.060
Scale factor point2	0.9996020	0.9999775
Distance for plane	3800.503	3801.948
Distance for Geo. Problems	3802.018	
Relative scale for distances	1/2510	1/54315

Table-4 Orebro city – comparison between UTM and Compound projection results

Ellipsoid parameters WGS 84 a= 6378137.00m. b=6356752.314m		
Distance from equator to standard parallel (Projection of compound) $X_0=6804494.7527$ m		
Scale factor for projections	0.9996	K1=-0.15 k2=1.15
Projections	UTM –Mercator  zone 33 , L=15°00' E	Projection of compound  standard parallel $B_0=61^\circ 21' N$  Center meridian L= 16° 30'E
Geographic coordinates lat. 3	N „00<15 59°	
Geographic coordinates log. 3	E „00<10 15°	
Triangular coordinates x3	6567902.082	6571284.415
Triangular coordinates y3	509505.876	673932.974
Scale factor point 3	0.9996011	0.9999830
Geographic coordinates lat. 4	N „00<19 59°	
Geographic coordinates log. 4	E „00<10 15°	
Triangular coordinates x4	6575325.764	6578709.501
Triangular coordinates y4	509487.311	674081.086
Scale factor point4	0.9996011	0.9999888
Distance for plane	7423.705	7426.563
Distance for Geo. Problems	7426.668	
Relative scale for distances	1/2506	1/70730



**Table-5 Malomo city – comparison between UTM and Compound projection results**

<b>Ellipsoid parameters WGS 84 a= 6378137.00m, b=6356752.314m</b>		
<b>Distance from equator to standard parallel (Projection of compound) X0=6804494.7527 m</b>		
<b>Scale factor for projections</b>	<b>0.9996</b>	<b>K1=-0.15 k2=1.15</b>
Projections	UTM –Mercator zone 33 , L=15°00' E	Projection of compound standard parallel $B_0=61^{\circ}21'N$ Center meridian L= $16^{\circ}30'E$
Geographic coordinates lat.1	55° 35' 43» N	
Geographic coordinates log.1	12° 56' 11» E	
Triangular coordinates x1	6162969.004	6169364.192
Triangular coordinates y1	619965.499	525542.987
Scale factor point 1	0.99981	0.9999952
Geographic coordinates lat.2	55° 35' 43» N	
Geographic coordinates log.2	13° 02' 00» E	
Triangular coordinates x2	6162791.691	6169057.787
Triangular coordinates y2	626073.380	531646.844
Scale factor point2	0.999790	0.99996
Distance for plane	6110.454	6111.543
Distance for Geo. Problems	6111.689	
Relative scale for distances	1/4950	1/41860

**Table-6 Malomo city – comparison between UTM and Compound projection results**

<b>Ellipsoid parameters WGS 84 a= 6378137.00m, b=6356752.314m</b>		
<b>Distance from equator to standard parallel (Projection of compound)</b> <b>X<sub>0</sub>=6804494.7527 m</b>		
<b>Scale factor for projections</b>	<b>0.9996</b>	<b>K1=-0.15 k2=1.15</b>
Projections	UTM –Mercator zone 33 , L=15°00' E	Projection of compound standard parallel B <sub>0</sub> =61° 21' N Center meridian L= 16° 30'E
Geographic coordinates lat.3	55° 34' 15» N	
Geographic coordinates log.3	12° 58' 15» E	
Triangular coordinates x3	6160185.227	6166536.36
Triangular coordinates y3	622056.125	527574.709
Scale factor point 3	0.9998	0.999976
Geographic coordinates lat.4	55° 37' 00» N	
Geographic coordinates log.4	12° 58' 15» E	
Triangular coordinates x4	6165284.859	6171632.623
Triangular coordinates y4	622205.197	527831.552
Scale factor point4	0.9998	0.999986
Distance for plane	5101.810	5102.731
Distance for Geo. Problems	5102.828	
Relative scale for distances	1/5013	1/52606

All tables show the distortions in distances measured by rectangular coordinates using Universal Transverse Mercator UTM zones (33, 34) and compound projections compared with distances measured from geodetic problems.

The best results were when the compound projection used. Because of the

position of Stockholm city is between zones 33 and 34, this made an overlapping, so that it has a weakness for rectangular coordinates when the UTM is used.

#### **4. CONCLUSION**

The coordinate system by compound projection with smart main systems better than of old coordinates systems by UTM for Sweden.

- Minimum distortions in distances were obtained by main compound projection and it was better than UTM.
- Errors in the compound projection for shorts distances 0.00- 20000.00 m  $\pm$  0.00 - 0.150 m; while in UTM  $\pm$ 0.00 – 10.00 m.
- The relative scale factor for the compound projection was better than in UTM at all cities without Sampson's correction.

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